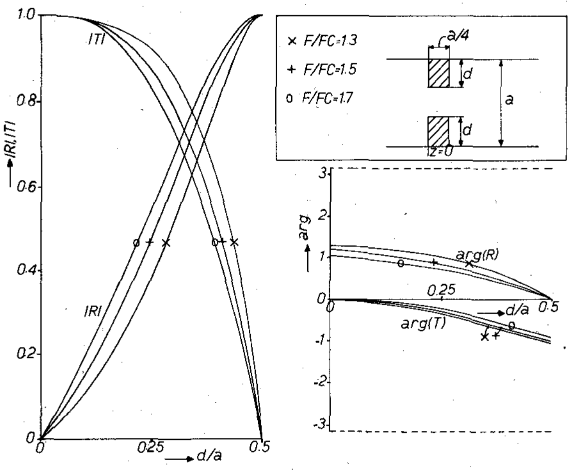

Fig. 2. Reflection and transmission factor of a thick diaphragm (incident $LSE_{0,1}$ mode = dominant mode).

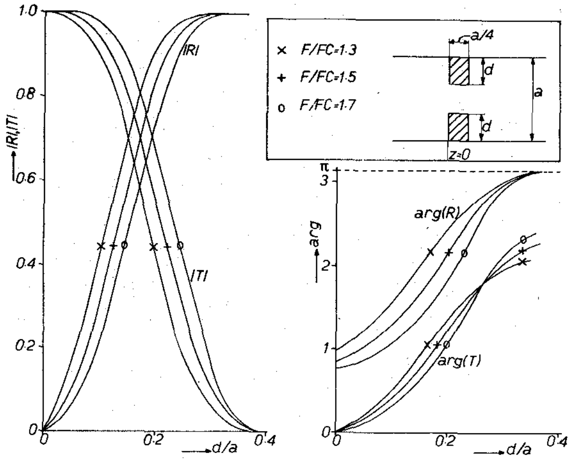

Fig. 3. Reflection and transmission factor of a thick diaphragm (incident $LSM_{1,0}$ mode = dominant mode). When $0.4 < d/a < 0.5$, $|T|$ is negligibly small.

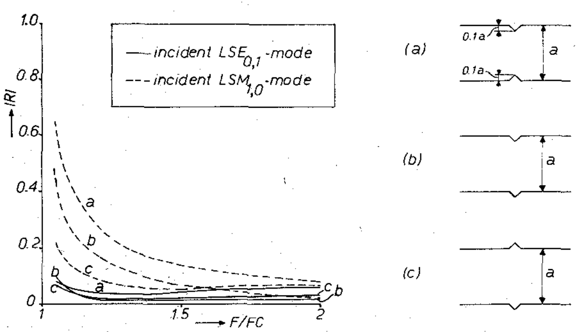

Fig. 4. Reflection factor of triangularly prismatic wall deformations in a rectangular waveguide.

REFERENCES

[1] I. A. Cermak and P. Silvester, "Solution of two-dimensional field problems by boundary relaxation," *Proc. Inst. Elec. Eng.* (*London*), vol. 115, pp. 1341–1348, 1968.
[2] P. Silvester and I. A. Cermak, "Analysis of coaxial line discontinuities by boundary relaxation," *IEEE Trans. Microwave Theory Tech.* (*Special Issue on Computer-Oriented Microwave Practices*), vol. MTT-17, pp. 489–495, Aug. 1969.
[3] I. A. Cermak and P. Silvester, "Boundary-relaxation analysis of rotationally symmetric electric field problems," *IEEE Trans. Power App. Syst.*, vol. PAS-89, pp. 925–932, May/June 1970.
[4] S. L. Richter, "Extension of boundary relaxation for finite-difference solution of field problems," *Proc. IEEE* (Lett.), vol. 58, pp. 1146–1148, July 1970.
[5] F. Sandy and J. Sage, "Use of finite difference approximations to partial differential equations for problems having boundaries at infinity," *IEEE Trans. Microwave Theory Tech.* (Corresp.), vol. MTT-19, pp. 484–486, May 1971.
[6] A. M. Patwari and J. B. Davies, "Scattering by infinite cylinders of arbitrary cross-section by the method of finite differences," *Electron. Lett.*, vol. 2, pp. 470–471, Dec. 1966.
[7] R. E. Collin, *Field Theory of Guided Waves.* New York: McGraw-Hill, 1960, pp. 225–229.
[8] G. De Jong, "Scattering by a perfectly conducting cylindrical obstacle in a rectangular waveguide," *Int. J. Electron.*, vol. 32, pp. 153–167, 1972.
[9] M. J. Beaubien and A. Wexler, "An accurate finite-difference method for higher order waveguide modes," *IEEE Trans. Microwave Theory Tech.* (*1968 Symposium Issue*), vol. MTT-16, pp. 1007–1017, Dec. 1968.
[10] G. E. Forsythe and W. R. Wasow, *Finite Difference Methods for Partial Differential Equations.* New York: Wiley, 1960, p. 198.
[11] R. S. Varga, *Matrix Iterative Analysis.* Englewood Cliffs, N.J.: Prentice-Hall, 1962, p. 59.
[12] S. D. Conte and R. T. Dames, "An alternating direction method for solving the biharmonic equation," *Math. Tables Other Aids Comput.*, vol. 12, pp. 198–205, 1958.

Finite-Gap Stripline-Latching Circulator

M. E. EL-SHANDWILY, MEMBER, IEEE, AND ESMAT A. F. ABDALLAH, STUDENT MEMBER, IEEE

***Abstract*—The stripline-latching circulator with finite gap is analyzed theoretically. The normalized resonant frequencies of the first-order modes and the normalized circulation frequencies are obtained numerically for different values of the gapwidth and dielectric constant of the ceramic filling the nonmagnetic gap.**

I. INTRODUCTION

Recently, Siekanowicz and Schilling [1] presented a theory for a three-port stripline-latching ferrite-junction circulator. The operation of the circulator is achieved by passing a pulse of direct current through a wire loop which is located between a ferrite cylinder and a concentric ferrite ring. The upper and lower portions of the ferrite ring and rod are in contact with ferrite disks. The circulator is switched by reversing the polarity of the current pulse.

The analysis of Siekanowicz and Schilling is an extension of Bosma's [2] and Fay and Comstock's [3] analyses for the stripline circulator. However, the effect of the nonmagnetic gap, in which the wire loop is located, has not been taken into consideration. In this short paper, we present the theory of the stripline-latching circulator with finite-nonmagnetic gap. The effect of the gapwidth and dielectric material on the circulator performance will be investigated.

II. ANALYSIS

The configuration of the stripline-latching circulator is shown in Fig. 1 [1]. In the following analysis, the circulator junction is divided into three regions; the ferrite post ($0 \le r \le r_1$), the nonmagnetic gap ($r_1 \le r \le r_2$), and the outer ferrite ring ($r_2 \le r \le r_3$). The electric fields in the three regions are

$$E_z^{\mathrm{I}} = J_n(x)(a_n e^{jn\phi} + a_{-n} e^{-jn\phi}), \qquad 0 \le r \le r_1 \tag{1}$$

$$E_z^{\mathrm{II}} = J_n(y)(b_n e^{jn\phi} + b_{-n} e^{-jn\phi}) + Y_n(y)(C_n e^{jn\phi} + C_{-n} e^{-jn\phi}), \qquad r_1 \le r \le r_2 \tag{2}$$

$$E_z^{\mathrm{III}} = J_n(x)(d_n e^{jn\phi} + d_{-n} e^{-jn\phi}) + Y_n(x)(f_n e^{jn\phi} + f_{-n} e^{-jn\phi}), \qquad r_2 \le r \le r_3 \tag{3}$$

Manuscript received February 5, 1973; revised June 22, 1973.
The authors are with the Electrical and Electronic Engineering Laboratory, National Research Centre, Dokki, Cairo, Egypt.

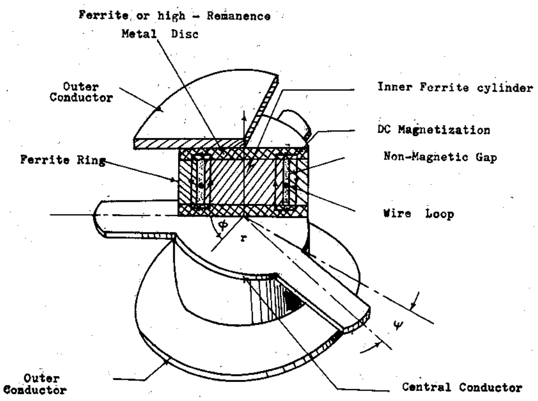

Fig. 1. Schematic representation of stripline-junction circulator.

where $a_{\pm n}, \cdots, f_{\pm n}$ are constants

$$x = \beta r$$
$$y = \beta_d r$$
$$\beta = \omega(\mu_0\epsilon_0\epsilon_f/\mu_{\text{eff}})^{1/2}$$
$$\beta_d = \omega(\mu_0\epsilon_0\epsilon_d)^{1/2}$$

and r_1, r_2, r_3 are the radii of the ferrite rod, inner, and outer radii of the ferrite ring.

The ϕ-component of the magnetic field is obtained from

$$H_\phi^{\text{I}} = \frac{j}{\omega\mu_0\mu_{\text{eff}}}\left(\frac{\partial E_z^{\text{I}}}{\partial r} + \frac{jK}{\mu r}\frac{\partial E_z^{\text{I}}}{\partial \phi}\right) \tag{4}$$

$$H_\phi^{\text{II}} = \frac{j}{\omega\mu_0}\frac{\partial E_z^{\text{II}}}{\partial r} \tag{5}$$

$$H_\phi^{\text{III}} = \frac{j}{\omega\mu_0\mu_{\text{eff}}}\left(\frac{\partial E_z^{\text{III}}}{\partial r} - \frac{jK}{\mu r}\frac{\partial E_z^{\text{III}}}{\partial \phi}\right). \tag{6}$$

The opposite sign of K in the second term on the right-hand side of (6) is due to the reversal of the dc magnetic field.

The boundary conditions at the outer surface of the ferrite post and the inner surface of the outer ferrite ring are used to express the constants $b_{\pm n}, \cdots, f_{\pm n}$. These boundary conditions are

$$\left.\begin{aligned} E_z^{\text{I}} &= E_z^{\text{II}} \\ H_\phi^{\text{I}} &= H_\phi^{\text{II}} \end{aligned}\right\} \quad \text{at } x = x_1 \quad y = y_1 \tag{7}$$

$$\left.\begin{aligned} E_z^{\text{II}} &= E_z^{\text{III}} \\ H_\phi^{\text{II}} &= H_\phi^{\text{III}} \end{aligned}\right\} \quad \text{at } x = x_2 \quad y = y_2 \tag{8}$$

where x_1, x_2, y_1, and y_2 are the values of x and y at r_1 and r_2. The remaining field components except H_r are 0. The above procedure gives the field components in all regions in terms of the constants a_{+n} and a_{-n}.

To determine the normal modes for the uncoupled case ($\psi = 0$), use is made of the following condition

$$H_\phi^{\text{III}} = 0 \quad \text{at } x = x_3. \tag{9}$$

The resonant frequencies of the normal modes are thus obtained as solutions of the following equation:

$$G_{n\pm}(x_3)[-N_{n\mp}(x_1,y_1)P_{n\pm}(x_2,y_2) + M_{n\mp}(x_1,y_1)Q_{n\pm}(x_2,y_2)] + L_{n\pm}(x_3)[N_{n\mp}(x_1,y_1)M_{n\pm}(x_2,y_2) - M_{n\mp}(x_1,y_1)N_{n\pm}(x_2,y_2)] = 0 \tag{10}$$

where

$$G_{n\pm}(x_i) = \left(\frac{\epsilon_f}{\epsilon_d\mu_{\text{eff}}}\right)^{1/2}\left[J_n'(x_i) \pm \frac{nK}{\mu x_i}J_n(x_i)\right]$$

$$L_{n\pm}(x_i) = \left(\frac{\epsilon_f}{\epsilon_d\mu_{\text{eff}}}\right)^{1/2}\left[Y_n'(x_i) \pm \frac{nK}{\mu x_i}Y_n(x_i)\right]$$

$$M_{n\pm}(x_i,y_i) = J_n(x_i)J_n'(y_i) - J_n(y_i)G_{n\pm}(x_i)$$
$$N_{n\pm}(x_i,y_i) = J_n(x_i)Y_n'(y_i) - Y_n(y_i)G_{n\pm}(x_i)$$
$$P_{n\pm}(x_i,y_i) = Y_n(x_i)J_n'(y_i) - J_n(y_i)L_{n\pm}(x_i)$$
$$Q_{n\pm}(x_i,y_i) = Y_n(x_i)Y_n'(y_i) - Y_n(y_i)L_{n\pm}(x_i).$$

The prime indicates differentiation with respect to the argument and $i = 1,2,3$. The upper sign refers to the positive mode while the lower sign refers to the negative mode.

The conditions for circulation are obtained by assuming that the device functions as a circulator. Following the procedure of Fay and Comstock [3] and carrying out the mathematical details, the device circulates when the following equation is satisfied.

$$F\left(x_1,x_2,x_3;y_1,y_2;\frac{K}{\mu}\right) = -F\left(x_1,x_2,x_3;y_1,y_2;-\frac{K}{\mu}\right) \tag{11}$$

where

$$\begin{aligned} F&\left(x_1,x_2,x_3;y_1,y_2;\pm\frac{K}{\mu}\right) \\ &= \{N_{1\mp}(x_1,y_1)[P_{1\pm}(x_2,y_2)G_{1\pm}(x_3) - M_{1\pm}(x_2,y_2)L_{1\pm}(x_3)] \\ &\quad - M_{1\mp}(x_1,y_1)[Q_{1\pm}(x_2,y_2)G_{1\pm}(x_3) - N_{1\pm}(x_2,y_2) \\ &\quad \cdot L_{1\pm}(x_3)]\}/\{N_{1\mp}(x_1,y_1)[Y_1(x_3)M_{1\pm}(x_2,y_2) - J_1(x_3)P_{1\pm}(x_2,y_2)] \\ &\quad - M_{1\mp}(x_1,y_1)[Y_1(x_3)N_{1\pm}(x_2,y_2) - J_1(x_3)Q_{1\pm}(x_2,y_2)]\}. \end{aligned} \tag{12}$$

In deriving (11) only the first-order mode ($n = \pm 1$) is taken into consideration. The preceding equations reduce to those of Siekanowicz and Schilling [1] by making $x_1 = x_2 = y_1 = y_2$.

III. Numerical Results and Discussion

Numerical solutions are obtained for the resonant frequencies of the first pair of the first-order modes [$n = +1, -1$ and the smallest x_3 that satisfies (10)] and for the circulation frequencies, for representative values of the parameters. In all the calculations the area of the ferrite rod is taken as equal to the area of the ferrite ring. Thus x_1 can be written in terms of x_3 as $x_1 = x_3/(1 + R^2)^{1/2}$ where $R = x_2/x_1$. Also $y = x(\epsilon_d/\epsilon_f\mu_{\text{eff}})^{1/2}$ and in the latching circulator μ_{eff} is given by

$$\mu_{\text{eff}} = 1 - \left(\frac{K}{\mu}\right)^2.$$

It is therefore possible to express all the arguments of Bessel's functions in (10) and (11) in terms of x_3, R, K/μ, and ϵ_f/ϵ_d.

First, we show the effect of the width of the nonmagnetic gap filled with ceramic having the same dielectric constant as the ferrite

TABLE I
NORMALIZED OUTSIDE RADIUS x_3 AT CIRCULATION $\epsilon_f/\epsilon_d = 1$

K/μ \ R	1	1.2	1.4	1.6
0	1.841184	1.841184	1.841184	1.841184
0.1	1.833664	1.833399	1.833253	1.833146
0.2	1.810994	1.809858	1.809219	1.808762
0.3	1.772828	1.769941	1.768312	1.767171
0.4	1.718509	1.712470	1.709060	1.706739
0.5	1.646937	1.635362	1.628906	1.624670
0.6	1.556325	1.534912	1.523388	1.516169

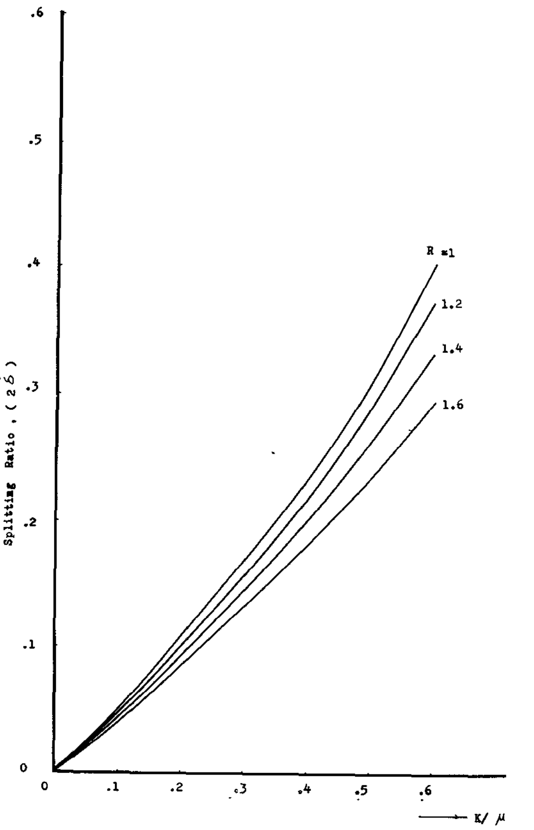

Fig. 2. The splitting ratio $2\delta' = (x_3^- - x_3^+)/x_{3\text{cir}}$ as a function of K/μ for different values of R and $\epsilon_f/\epsilon_d = 1$.

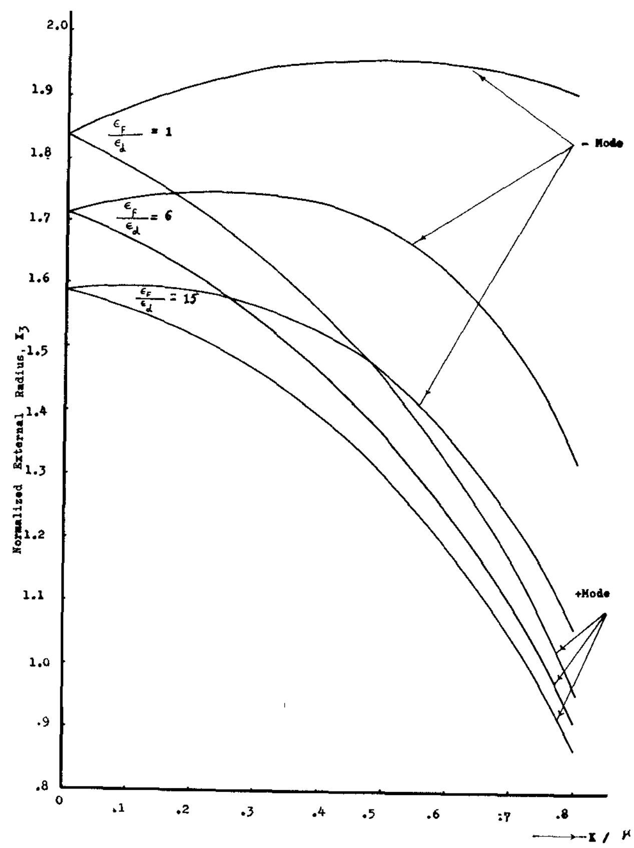

Fig. 3. Normalized external radii for the first-order–first-pair resonant modes as a function of K/μ, for $R = 1.1$ and different values of ϵ_f/ϵ_d.

($\epsilon_f/\epsilon_d = 1$). The normalized outside radius x_3 at circulation is shown in Table I for $\epsilon_f/\epsilon_d = 1$ as a function of K/μ and R. Fig. 2 gives the ratio $(x_3^- - x_3^+)/x_{3\text{ cir}}$ which is proportional to the bandwidth [1], [3]. The case for which $R = 1$ corresponds to the solutions of Siekanowicz and Schilling [1] in which the gapwidth is neglected. The results shown in Table I indicate that the external normalized radius x_3, to a good degree of approximation, is independent of the gapwidth as specified by the ratio R. The bandwidth decreases by increasing the ratio R. However, in the typical range of design $(0.2 \le K/\mu \le 0.5)$, the reduction in bandwidth is small. It should be noted that similar results have been obtained by the authors for the latching H-plane waveguide Y-junction circulator [5].

The volume of the ferrite material is proportional to $x_3^2/1 + R^2$. Since x_3 is almost independent of R, then increasing R decreases the ferrite volume. This can be of some value in reducing the insertion loss, if the dielectric ring material is less lossy than the ferrite material. Also, the switching energy, which is proportional to the ferrite volume, is reduced by increasing R, without affecting the circulation frequency. The percentage saving of switching energy when using nonmagnetic gap as compared with zero gap is equal to $100\ (R^2-1)/(R^2+1)$, which reaches about 38 percent for $R = 1.5$. However, the bandwidth is slightly decreased.

In order to see the effect of the permittivity of the dielectric ring material on the circulator performance several solutions of (10) and (11) have been obtained. Sample calculations for the resonant frequencies of the first-pair–first-order modes are shown in Fig. 3, for $R = 1.1$ and different values of ϵ_f/ϵ_d. Fig. 4 shows the normalized external radius x_3 for circulation as a function of K/μ for the same value of R as in Fig. 3 and different values of ϵ_f/ϵ_d. It is clear from these two figures that the external normalized radius for circulation lies between the normalized radii of the positive and negative modes. Also, x_3 for circulation decreases by increasing the ratio ϵ_f/ϵ_d.

The ratio $(x_3^- - x_3^+)/x_{3\text{ cir}}$ which is proportional to the bandwidth, is shown in Fig. 5 for the same parameters as in Fig. 4. From this figure it is seen that the bandwidth decreases by changing the dielectric constant of the ceramic ring material from that of the ferrite material. However, for $\epsilon_f/\epsilon_d \le 2$ the change in the circulator bandwidth is very small, which means that in this range the bandwidth is not sensitive to changes in the dielectric constant of the ceramic ring material. The case of $\epsilon_f/\epsilon_d = 15$ (for $\epsilon_f = 15$ this corresponds to air ring) gives the smallest bandwidth and the smallest normalized external radius. For a given frequency of operation this case gives the smallest circulator.

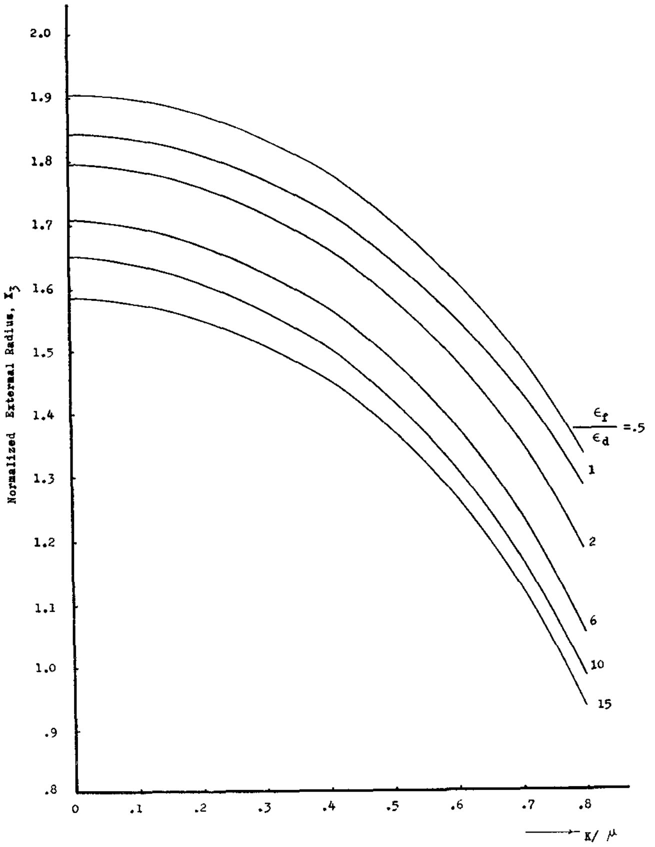

Fig. 4. Normalized external radius x_3 for circulation as a function of K/μ for $R = 1.1$ and different values of ϵ_f/ϵ_d.

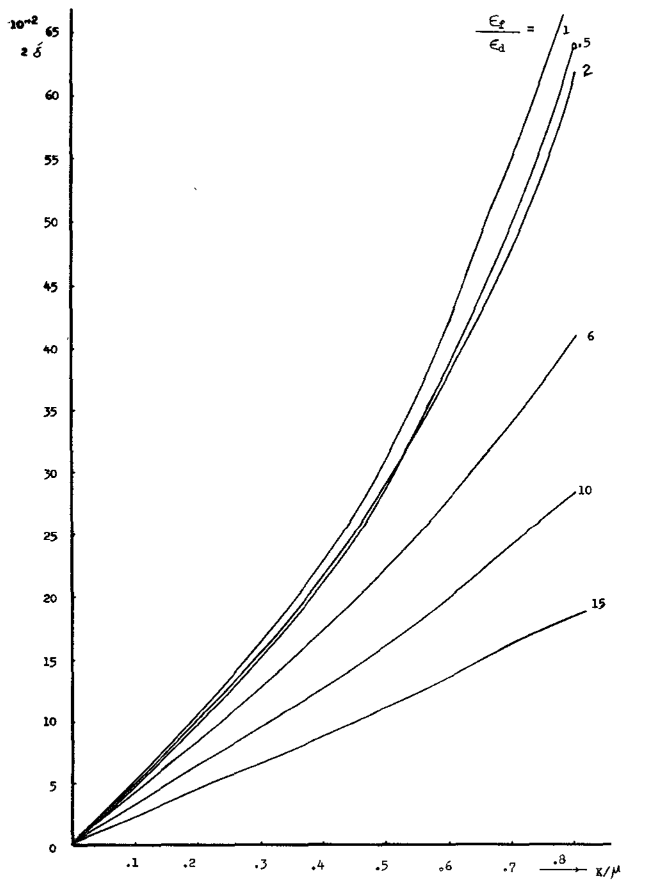

Fig. 5. The splitting ratio $2\delta' = (x_3^- - x_3^+)/x_{3_{0ir}}$ as a function of K/μ for $R = 1.1$ and different values of ϵ_f/ϵ_d.

IV. Conclusions

The effect of the width and the dielectric material of the nonmagnetic gap on the operation of latching stripline circulators has been studied. Numerical results show that in the typical range of design the external normalized radius and the bandwidth are slightly affected by the gapwidth. Generally speaking the bandwidth decreases by changing the dielectric constant of the ceramic ring from that of the ferrite. The normalized external radius decreases by decreasing the dielectric constant of the ceramic ring. The bandwidth is not sensitive to the variations in ϵ_f/ϵ_d up to the value 2.

References

[1] W. W. Siekanowicz and W. A. Schilling, "A new type of latching switchable ferrite junction circulator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 177–183, Mar. 1968.
[2] H. Bosma, "On stripline Y-circulation at UHF," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, pp. 61–72, Jan. 1964.
[3] C. E. Fay and R. L. Comstock, "Operation of the ferrite junction circulator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 15–27, Jan. 1965.
[4] W. W. Siekanowicz, R. W. Paglione, and T. E. Walsh, "A latching ring-and-post ferrite waveguide circulator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 212–216, Apr. 1970.
[5] M. E. El-Shandwily, A. A. Kamal, and E. A. F. Abdallah, "General field theory treatment of *H*-plane waveguide junction circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 392–403, June 1973.

Curved-Rim Open Resonators

A. CONSORTINI

***Abstract*—Field configurations and resonant frequencies are analytically derived for some low-loss modes of a Fabry–Perot (FP) open resonator having curved rims along the edges. Since the low-loss modes are limited by a caustic surface, the problem can be simply treated by neglecting the diffraction due to the finite dimensions of the mirrors. The results are compared with those obtained by numerically solving the integral equation of the open cavity.**

I. Introduction

As is well known, the losses of a class of open resonators are so low that field configurations and resonant frequencies can be obtained, with a good approximation, without taking into account the effects of the diffraction due to the finite dimensions of the mirrors. In general, this class includes those open cavities whose low-loss modes are limited by a caustic surface [1]. Typical examples are the curved stable resonators [1], [2], the so-called flat-roof resonator [3], the quasi-corner resonator [4], and some types of rimmed resonators [5], [6].

Another type of cavity, where the low-order modes are expected to be represented by fields bounded by a caustic surface, is that represented in Fig. 1 constituted by a Fabry–Perot (FP) resonator having curved rims along the edges. In the present short paper, mode configuration and resonances of it are determined by neglecting the diffractional effects.

As usual, the problem is reduced to the investigation of the infinite-strip case. The curved-rim sections are assumed to join the flat portion of the mirrors continuously (Fig. 1).

II. Theory

With reference to rectangular coordinates x,y,z with the origin at the center of the cavity, Fig. 1, the problem is to find a solution of the wave equation, for instance, the electric field E parallel to y, which satisfies the boundary conditions on the mirrors. We will

Manuscript received March 3, 1973; revised June 21, 1973.
The author is with the Istituto di Ricerca sulle Onde Elettromagnetiche, Firenze, Italy.